

\mathbb{R}^3

$$f(x, y, z) = ((x+y+z), -3(x+y-z), -2(x+y-z))$$

$$f(x, y, z) = (x+y-z) \cdot (1, -3, -2) \quad \forall (x, y, z)$$

\Rightarrow

$$\boxed{\text{Im} f = \langle (1, -3, -2) \rangle}$$

en effet

$$\begin{aligned} f(e_1) &= (1, -3, -2) \\ f(e_2) &= (1, -3, -2) \\ f(e_3) &= (-1, 3, 2) \end{aligned} \quad \left| \begin{array}{l} \neq \\ \Rightarrow f(e_3) = -f(e_1) = -f(e_2) \end{array} \right.$$

$$\text{Ker} f = \{ (x, y, z) \in \mathbb{R}^3 \mid f(x, y, z) = (0, 0, 0) \}$$

$$f(x, y, z) = (0, 0, 0) \Rightarrow (x+y-z) \cdot (1, -3, -2) = (0, 0, 0)$$

$$(1, -3, -2) \neq (0, 0, 0) \quad \text{alors} \quad \underline{\underline{x+y-z=0}} \Rightarrow$$

$$\underline{\underline{z = x+y}}$$

$$\Rightarrow f(x, y, z) = (0, 0, 0) \Rightarrow (x, y, z) = (x, y, x+y)$$

$$(x, y, z) = (x, 0, x) + (0, y, y) = x \underbrace{(1, 0, 1)} + y \underbrace{(0, 1, 1)}$$

$$\Rightarrow \underline{\underline{\text{Ker} f = \langle (1, 0, 1), (0, 1, 1) \rangle}}$$