$x f(e_1) + y f(e_2) + z f(e_3) = 0_{\mathbb{R}^3}$

$-4y f(e_1) + y f(e_2) + 3y f(e_3) = 0_{\mathbb{R}^3}$ $\forall y$

$y(-4 f(e_1) + f(e_2) + 3 f(e_3)) = 0_{\mathbb{R}^3}$

$\Rightarrow f(e_2) = 4 f(e_1) - 3 f(e_3)$

$$\begin{pmatrix}1\\2\\3\end{pmatrix} = 4\begin{pmatrix}1\\-1\\0\end{pmatrix} - 3\begin{pmatrix}1\\-2\\-1\end{pmatrix} = \begin{pmatrix}4\\-4\\0\end{pmatrix} + \begin{pmatrix}-3\\+6\\+3\end{pmatrix} = \begin{pmatrix}4-3\\6-4\\0+3\end{pmatrix}$$

$\Rightarrow \operatorname{Im} f = \{f(e_1), f(e_3)\}$.

$\operatorname{Im} f = \langle f(e_1), f(e_3)\rangle \Rightarrow \dim \operatorname{Im} f = 2$.

on vérifie bien $\checkmark$

$\dim(\mathbb{R}^3) = \dim(\operatorname{Ker} f) + \dim(\operatorname{Im} f)$

$3 = 1 + 2$.

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Ex6 :

$$A = \begin{pmatrix}1 & 1 & -1\\-3 & -3 & 3\\-2 & -2 & 2\end{pmatrix}.$$

$f : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$

$u = (x, y, z) \longrightarrow V = f(x, y, z) = (X, Y, Z)$.

$\cancel{B} = u^t = \begin{pmatrix}x\\y\\z\end{pmatrix} \longrightarrow \cancel{Q} = V^t = \begin{pmatrix}X\\Y\\Z\end{pmatrix} = A\begin{pmatrix}x\\y\\z\end{pmatrix}$

Voir

$$\begin{pmatrix}X\\Y\\Z\end{pmatrix} = \begin{pmatrix}1 & 1 & -1\\-3 & -3 & 3\\-2 & -2 & 2\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}x+y-z\\-3x-3y+3z\\-2x-2y+2z.\end{pmatrix}$$ Voir chap III

$\Rightarrow f(x, y, z) = (X, Y, Z) = (x+y-z, -3x-3y+3z, -2x-2y+2z)$

$f(x, y, z) = (x+y-z, -3x-3y+3z, -2x-2y+2z)$