$$\begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \end{array}\begin{array}{c} \begin{array}{ccc} f(e_1) & f(e_2) & f(e_3) \end{array} \\ \begin{pmatrix} 1 & 1 & 1 \\ -1 & 2 & -2 \\ 0 & 3 & -1 \end{pmatrix} \end{array}$$

$e_1 = (1,0,0)$ $\quad e_2 = (0,1,0)$ $\quad e_3 = (0,0,1)$

$f(e_1) = (1,-1,0)$ $\quad f(e_2) = (1,2,3)$ $\quad f(e_3) = (1,-2,-1)$

Voir chap II $\quad$ or $(x,y,z) = x e_1 + y e_2 + z e_3$.

$$f(x,y,z) = x f(e_1) + y f(e_2) + z f(e_3).$$

$$f(x,y,z) = x(1,-1,0) + y(1,2,3) + z(1,-2,-1)$$

$$= (x,-x,0) + (y,2y,3y) + (z,-2z,-z)$$

$$= (x+y+z, -x+2y-2z, 3y-z).$$

$$\text{Ker} f = \{(x,y,z) \in \mathbb{R}^3 / f(x,y,z) = (0,0,0)\}$$

$$f(x,y,z) = (0,0,0) \Rightarrow \begin{cases} x+y+z = 0 \\ -x+2y-2z = 0 \\ 3y-z = 0 \end{cases} \quad \begin{cases} x+y+3y = 0 \\ -x+2y-6y = 0 \end{cases} \Rightarrow z = 3y.$$

$$\Rightarrow \begin{cases} x+4y = 0 \\ -x+4y = 0 \end{cases} \quad x+4y = 0 \Rightarrow x = -4y.$$

$$f(x,y,z) = (0,0,0) \Rightarrow (x,y,z) = (-4y, y, 3y) = y(-4,1,3).$$

$$\Rightarrow \text{Ker} f = \langle (-4,1,3) \rangle. \quad \dim \text{Ker} f = 1.$$

$$* \ \text{Im} f = \{v \in \mathbb{R}^3 / v = f(u) \ \forall u \in \mathbb{R}^3\}$$

$$\text{Im} f = \{f(e_1), f(e_2), f(e_3)\} \quad \text{(mat des vect L.I.)}$$

or d'après ci dessus

$f(x,y,z) = x f(e_1) + y f(e_2) + z f(e_3) = (0,0,0) \Rightarrow x = -4y, \ z = 3y \quad \forall y \in \mathbb{R}$

$\Rightarrow \{f(e_1), f(e_2), f(e_3)\}$ n'est pas libre.